

Textos para Discussão

Sovereign Debt Risk Modeling and Portfolio Management

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Abstract

This paper presents a compilation of interesting models to treat sovereign debt portfolio applied to debt management offices. It starts with an analytical balance sheet and net worth optimization and evolves onto policy decisions based on deterministic and stochastic debt simulation models. Important risk measures derived from Value-at-Risk variations are drawn from these, which enables debt managers to prospect the results of a given funding strategy. Finally, further analysis on the asset side are introduced in light of the shape and size of government liabilities to verify if policy decisions change in such circumstances. The conclusion is that while balance sheet and net worth optimization are more affine to optimal taxation theory, debt service simulation models are more appealing to most practitioners.

JEL Codes: E61, G11, H63

Keywords: balance sheet, debt strategy, maturity structure, interest and currency risk, net worth, debt simulation models.

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Acronyms

ALM	Asset and Liability Management
BCB	Brazilian Central Bank
BoP	Balance of Payments
BSA	Balance Sheet Approach
CDS	Credit Default Swaps
CIR	Cox, Ingersoll and Ross interest rate model
CKLS	Chan, Karolyi, Longstaff and Sanders model
CL	Contingent Liabilities
CNW	Consolidated Net Worth
CPI	Consumer Price Index
DMO	Debt Management Office
DSA	Debt Sustainability Analysis
DPA	Debt Portfolio Analysis
GAAP	General Accepted Accounting Principles
GBM	Geometric Brownian Motion
GFS	Government Finance Statistics
GGGD	Gross General Government Debt
IMF	International Monetary Fund
MoF	Ministry of Finance
NPSD	Net Public Sector Debt
RNW	Reported Net Worth
SDDS	Special Dissemination Data Standard
SOE	State Owned Enterprises
UNCTAD	United Nations Conference on Trade and Development
VaR	Value-at-Risk
WB	The World Bank
YTM	Yield to Maturity

1- Introduction

This paper will introduce risk modeling tools capable of helping debt managers to measure their exposure to certain risks originated on their portfolio. First, it will take advantage from the BSA² discussion held in many articles [Proite,(2014)], [Das et al,(2012b)], [Allen et al,(2002)], to unfold it into more elaborated ALM³ risk models considering optimality in the balance sheet, its implications on debt sustainability and the government's investment decisions of its assets to protect the budget from permanent effects coming from macroeconomic shocks. Second, debt service simulation models are discussed under an ALM context. While presenting the tools commonly used to treat debt portfolio risk, it will discuss the applicability of these to developing economies.

Having that said, it will be observed that institutional set up and hindered market access may represent meaningful restrictions to some countries to build on these frameworks towards more analytical models such as long term optimal benchmark. Nevertheless, the discussion will follow a step-by-step approach that would shed some light on specific types of risk exposure inherent for most debt portfolios. Such discussion could be used to evaluate the functionality of the financial system and local capital market, currency and maturity composition of external debt and optimal reserve accumulation.

Some articles have pointed that BSA approach holds a few shortcomings [IMF, (2004)]. To begin with, it should be seen as a conceptual framework to oversee the types of exposure a given sovereign is bounded to, assessing its vulnerabilities and related policy options, given the prevailing political and economic environment. Policy makers should keep in mind that the BSA is useful to minimize financial risks while adjusting imbalances on the economic structure. Additionally, it is a static tool and it is hard to model it over time if one consider the characteristics of most relevant assets and liabilities. Considering those, it would be valuable to move onto models that would enhance risk management and allow the decision making process to be more elaborated.

² Balance Sheet Approach

³ Asset and Liability Models

Section 2 discusses an analytical approach to the balance sheet and how they could be translated in policy decisions related to debt portfolio management. Section 3 introduces debt simulation models divided in deterministic and stochastic techniques which are widely used by debt managers and are closer to more advanced debt portfolio benchmarking models. Important measures are derived from that. Section 4 adds the asset side in the analysis and describes if the liability side reacts to that interference and section 5 concludes.

2- **Balance Sheet Analytical Framework**

2.1- Optimal Balance Sheet Management

During the 90's, a great deal of countries managed to control their debt size and risk levels and strengthened their fiscal positions. With that, attention was brought on broader balance sheet liabilities such as pension funds obligations and state owned enterprises (SOE) support schemes provided by the federal government. In different ways, countries like Australia, New Zealand and Sweden have developed an institutional set up to scan other liabilities arising from the public sector. The Swedish National Debt Office, for instance, has a team dedicated to monitor liabilities that may crystalize in their balance sheet and elaborate plans to finance those without pressuring market conditions.

Discussions have been held on how to best structure government's balance sheet management to reduce overall financial risk. Au-Yeung et al (2006) propose an optimal balance sheet approach and estimate types of assets and liabilities capable of reducing financing risk for the public sector.

This framework uses a balance sheet extracted from a GFS⁴ mold, that is, putting in perspective assets and liabilities and defining net worth as a residual from them. Nonetheless, some assets are not included mainly due to valuation problems arising from the government side and its prime asset: the power to tax. Considering the

⁴ General Finance Statistics - IMF

problems from measuring certain public assets, government balance sheet management requires a different framework for determining optimal investment strategies.

A government financial portfolio can be structured to reduce permanent impacts of macroeconomic shocks and its effects on the tax base. Indeed, the relationship between taxation and balance sheet assets and liabilities is central to the economic literature [Barnhill and Kopits, (2003)].

First, the model's intuition could be grasped from understanding an important conceptual tool for analyzing government balance sheet: the intertemporal budget constraint. This budget constraint requires that at any date, the sum of net worth and the net present value of taxation to be equal to the net present value of government spending. In this way, the intertemporal budget constraint relates the government balance sheet in any period to the contingent asset and liabilities that can affect the balance sheet. If current government spending is higher than current taxation, the government can issue debt (or some other liability). However, this simply means taxes need to be higher sometime in the future, with potential implications on optimal taxation.

For a macroeconomic shock that temporarily reduces economic growth, the government could resort to deficit financing by selling financial assets or issuing debt. However, if a non-anticipated shock lead to a permanent change in the resources available to government (for example, a fall in the present value of taxation revenue), the government would need to adjust fiscal policy because deficit financing would not be sustainable. Alternatively, governments could attempt to structure their financial portfolios to hedge against such risks [Au-Yeung et al, (2006)]. Specifically, an effective budget hedge would result in a positive financial return for the government when tax collection fades during a shock⁵. Therefore, this framework suggests that the optimal portfolio for a country depends on the structure of the economy. Apparently for practical reasons, it is extremely difficult to correctly evaluate and scrutinize the types of shocks an economy will be subjected to in the future.

⁵ See the 2008-9 crisis effect on Brazil. Because the Public Sector was long in foreign currency, external shocks translated into relatively weaker currency resulted in lower net debt to GDP ratio, even when economic activity contracted.

Formally, the model has a set up where individuals are assumed to be infinitely lived, risk neutral, and maximize the expected utility derived from all future consumption:

$$U_t = E_t \sum_{j \geq 0} \rho^j c_{t+j} \quad (1)$$

where ρ is a discount factor, and c_{t+j} is consumption in period $t + j$. Individuals receive income Y_{t+j} and pay taxes on that at a rate τ_t . As taxes are distortionary there is an excess burden of taxation denoted by a convex loss function $h(\tau_t)$. Individuals are also able to trade a given set of assets, so that the individual budget constraint is given by:

$$c_t + \sum_k p_{t,k} A_{t,k} = Y_t [1 - \tau_t - h(\tau_t)] + \sum_k (p_{t,k} + f_{t,k}) A_{t-1,k} \quad (2)$$

where $A_{t,k}$ is the quantity of asset k held at the end of period t ; $p_{t,k}$ is the price of asset k (denoted in terms of consumption goods); and $f_{t,j,k}$ is the stream of cash flows derived from holding asset k . Individual maximization implies that expected returns across assets are equal, that is $E_t(1 + r_{t+1,k}) = \frac{1}{\rho}$ for all k , where

$r_{t+1,k} = \frac{(p_{t,k} + f_{t,k})}{p_{t,k}} - 1$. This means that expected returns are inversely proportional from the discount factor, a strong assumption which will simplify the optimization program.

The government budget constraint is defined by

$$\tau_t Y_t + \sum_k (p_{t,k} B_{t,k}) = G_t + \sum_k (p_{t,k} + f_{t,k}) B_{t-1,k} \quad (3)$$

Where $\tau_t Y_t$ are tax revenues and $B_{t,k}$ represents financing coming from debt creation, to finance government expenditure, $G_{t,k}$ (treated as exogenous in this model), and to face debt service. For any given financing instrument k , the government can be a net debtor or net lender. Because of that, $B_{t,k}$ could either assume positive or negative values (net debt). Substituting equations (3) and (2) in (1) gives:

$$U_t = E_t \sum_{j \geq 0} \rho^j \{Y_{t+j} [1 - h(\tau_{t+j})]\} \quad (4)$$

The government chooses an optimal tax rate and debt portfolio to maximize individual utility (4) subject to its own budget constraint (3). In effect, the government's objective is to choose the structure of taxes and debt that minimize the expected present value of

the excess burden of $h(\tau_t)$, that is, minimize the welfare loss coming from distortionary taxation. The first-order conditions are:

$$E_t[h'(\tau_{t+1})] = h'(\tau_t), \text{ for } k = 0 \text{ and}$$

$$\rho E_t[h'(\tau_{t+1})(1 + r_{t+1,k})] = h'(\tau_t), \text{ for } k > 0 \quad (5),$$

Where $k = 0$ represents the case where individuals are equal off in different periods (possess a riskless asset). In other words, optimality requires expected marginal excess burden of taxation to be constant through time.

If second-order conditions are satisfied, the deadweight loss τ_t is $h(\tau_t) = \left(\frac{h}{2}\right)\tau_t^2$, then from the first order conditions that an optimal policy requires

$$COV_t(\tau_{t+1}^*, r_{t+1,k}^*) = 0 \quad (6)$$

Where $\tau_{t+1}^* = \tau_{t+1} - E(\tau_{t+1})$ is the unexpected component of tax rate and $r_{t+1,k}^*$ is the up-to-date return of asset k . These terms reflect the unanticipated components of changes in tax rates or returns, which represent a random walk.

The draw line here is that equation (6) states that if the covariance between innovations in the tax rate and returns, for a specific debt, is negative then the government could improve tax-smoothing by issuing more of this form of debt. The converse is also true: if the covariance is positive then the government could improve tax smoothing by purchasing more of this form of debt [Au-Yeung et al, (2006)]. The main conclusion is that the government should smooth tax rates across different states of the world, as well as over time.

To estimate the government portfolio it is necessary to introduce an equation describing how new taxation arises through time:

$$\tau_{t+1}^* = (1 - \varphi)e^{-\dot{y}}[\sum_k r_{t+1,k}^* d_{t,k} + \sum_{j \geq 0} \rho^j g_{t+1,j}^*] - \tau_t \sum_{j \geq 0} \varphi^j y_{t+1,j}^* \quad (7)$$

Where y_t is the growth rate and \dot{y} is its mean value. $\sum_{j \geq 0} \varphi^j y_{t+1,j}^*$ ⁶ is the present value of future growth rates of real output, where φ is the discount factor. Note that $y_{t+1,j}^*$ describes unexpected changes in government output and its policy reaction to

⁶ $y_{t+1,j}^* = E_{t+1}y_{t+1+j} - E_t y_{t+1+j}$, denoted as innovations.

raise tax at a constant rate (φ). Analogously, $\sum_{j \geq 0} \rho^j g_{t+1,j}^*$ is the present value of future government expenditures relative to output⁷. The ratio of security k to output is denoted by $d_{t,k}$. Equation (7) can be seen as government needs in terms of tax revenues expressed in present value to face debt stock and the spending flow discounted at a given rate. That is, tax rates will need to adjust whenever there are *unexpected* changes in the value of government debt, government spending or output growth. For a government that is already optimally managing the balance sheet, the current tax rate already incorporates anticipated obligations [Au-Yeung et al, (2006)]. Doing (7) in (6), one may observe the optimality condition for a given government security:

$$\begin{aligned} \sum_l COV_t(r_{t+1,k}^*, r_{t+1,l}^*) d_{t,l} + COV_t(r_{t+1,k}^*, \sum_{j \geq 0} \rho^j g_{t+1,j}^*) - \\ w_t COV_t(r_{t+1,k}^*, \sum_{j \geq 0} \varphi^j y_{t+1,j}^*) = 0 \end{aligned} \quad (8)$$

Where $w_t = \left[\frac{e^y}{(1-\varphi)} \right] \tau_t$ is a weighting factor.

That is, the government can smooth taxes to balance non-anticipated shocks in the present value of government spending and output through the issuance or purchase of state contingent securities. Au-Yeung et al (2006) focus on shocks that affect the present value of output growth, assuming that the covariance between innovations in the present value of government spending and returns on assets (the second term in equation (8)), is zero. To find a solution to the government's optimal portfolio, one must impose:

$$d = w_t \Sigma_r^{-1} \Sigma_{y,r} \quad (9)$$

Where Σ_r is the non-singular variance-covariance matrix of returns and $\Sigma_{y,r}$ is the variance-covariance matrix of returns and output growth.

To solve (9), one needs to calculate innovations in returns and the present value of future rates of growth in real output. To simplify, a dual choice is imposed to consider the optimal share of local bonds versus external bonds (long term)⁸. The return on domestic debt, $r_{t+1,d}$, is influenced by the domestic nominal interest rate l_{t+1} ,

⁷ $g_{t+1,j}^* = (E_{t+1} G_{t+1+j} - E_t G_{t+1+j}) / Y$

⁸ This could be extended to other asset classes, such as equities.

movements on market interest rate (long term), and domestic inflation π_{t+1} . Foreign long-term debt returns, $r_{t+1,f}$ is affected by the change in the exchange rate Δs_{t+1} . New returns for each type of bonds are given by:

$$r_{t+1,d}^* = - (l_{t+1,d} - E_t l_{t+1,d}) - (\pi_{t+1} - E_t \pi_{t+1})$$

$$r_{t+1,f}^* = - (l_{t+1,f} - E_t l_{t+1,f}) - (\pi_{t+1} - E_t \pi_{t+1}) + (\Delta s_{t+1} - E_t \Delta s_{t+1}) \quad (10)$$

To calculate these innovations in real returns, vector-autoregressive (VARs) estimations are used to formulate expectations for the inflation rate, the percentage change in the exchange rate, the long-term domestic interest rate and the long-term foreign interest rate. Because expectations at time t depend only on information available up to time t, a new VAR must be estimated for each time period. The same method is used to calculate unanticipated changes in the output real growth.

Finally, Au-Yeung et al (2006) have applied this model to Australian debt data and derived a variance-covariance matrix where deviations from domestic and external mean returns are negatively correlated to GDP growth. This result is consistent with the literature (for example, Missale (1999)). Specifically, the results show that it is optimal for the government to invest a relatively larger amount in domestic rather than foreign debt. This is largely driven by the volatility in the exchange rate. Volatility in the exchange rate (and therefore in foreign returns) is not necessarily bad, provided unexpected depreciation varies positively with innovations⁹ in output.

The model presented above describes the inter-relationship between individual inter-temporal decisions on consumption and attached utility, the government budget constraint to finance its spending and debt obligations, and then introduces the effects of those decisions on welfare loss in order to minimize it throughout time. With that, the government would balance its tax levels and portfolio decisions to manage unanticipated macroeconomic shocks in the present value of spending and output. By doing that, debt should be structured so as to provide a hedge against macroeconomic shocks to the budget. The idea is that changes in the present value of government

⁹ Defined as deviations from the mean

revenues and expenditures can be hedged by debt returns in order to smooth tax rates, which is consistent with ALM purposes.

2.2- Net Worth and Policy Implications

Another interesting work related to the sovereign balance sheet concentrate the analysis on targeting the concept of net worth, by using conventional accounting measures to forecast future spending and tax revenues, estimating a path rather than a net worth level. This section discusses how an intertemporal sovereign budget constraint could be used to model a broad concept of net worth helping to determine long term fiscal settings.

Bradbury et al (1997) relied on the fact that New Zealand pioneered as the first sovereign to fully adopt GAAP¹⁰ reporting to the national accounts statistics. Allegedly, they have benefited from greater reliability on the indicators allowing better financial decision making by the government and reducing inefficiencies related to fiscal illusion.

With that, it is possible to write a government budget, for a given period t as:

$$B_t = T_t - (G_t + H_t + \delta A_{t-1} + iD_{t-1}) + \pi_t + V_t + \Delta SSB + (fx_t + fx_{t-1})FD_t + \varphi_t^{SOE} \quad (11)$$

Where

B_t : Operational Balance	π_t : Net profit of Assets
T_t : Non-financial Revenues	V_t : Valuation of Commercial Forests
G_t : Government Expenditures	ΔSSB : Social Security Balance Changes
H_t : Subsidies and Transfers	$fxFD_t$: Net gains/losses on Foreign Exchange denominated debt
δA_{t-1} : Depreciation of physical assets	φ_t^{SOE} : Net dividends/losses from State Owned Companies
iD_{t-1} : Net Interest paid on the Net Debt	

¹⁰ General Accepted Accounting Principles

A few notes must be made. It is unusual from the sovereign balance sheet point of view to incorporate non-financial or physical assets in the analysis¹¹. Second, despite total government debt has been considered as a part of spending, emphasis is given to external debt, which is explicitly denoted in (11) because the disturbing effects oscillating exchange rate could spur into the budget.

It is important to express financial sustainability in terms of the relationship between the budgetary aggregates, and whether the projected cash outflows can be financed by the projected cash inflows under current policy settings. For that purpose, the intertemporal budget must be considered. Before that, for any period, the budgetary financial identity holds for any given period t , that is, outstanding debt variation depends on the level of government spending, non-financial revenues net of subsidies and debt service.

$$\Delta D_t = G_t - (T_t - H_t) + iD_{t-1} \quad (12)$$

or $\Delta D_t = -P + iD_{t-1}$, where P is the Primary Balance

Next, the concept of Report Net Worth (RNW) is presented as the fiscal position at time t , and should reflect the GAAP based estimate of the residual of the balance sheet reflecting the transactions occurred in the previous period, that is¹² :

$$\Delta RNW = RNW_t - RNW_{t-1} = B_t \quad (13)$$

The problem of the RNW it is a backward looking measure of the accounting value of assets less the value represented by liabilities - it records the financial effects of transactions that have occurred up until the reporting date and which satisfy the reliability criteria. Because there are uncertainties coming from unknown future events, a different indicator is devised. Comprehensive net worth (CNW) is a forward looking measure, which includes RNW as an opening balance (after adjustment for asset value performance), but which also takes into account future cash flows under current policy settings, and therefore captures sovereign wealth in a more satisfactory manner [Bradbury et al, (1997)]. The two net worth measures (reported net worth and

¹¹ See IMF (2004).

¹² Originally, Bradbury et al (1997) suggest $\Delta RNW = RNW_t - RNW_{t-1} = B_t + \text{physical assets}$. Because of the discussion on practical reasons held in section 2, physical assets will be disregarded without harming the analysis.

comprehensive net worth) focus on different things and have different implications for decision making. CNW is defined as:

$$CNW_t = RNW_{t-1} + \sum_{t=1}^{\infty} \left(\frac{(P_t - iD_{t-1}) \pm \alpha_t}{(1+i)^t} \right) \quad (14)$$

Where α_t is a term for unknown variables and CNW_t is the Comprehensive Net Worth.

In a sense, CNW translates the idea of ALM, as it incorporates assets and liabilities for the overall public sector and relates them to the dynamic position of the budget. Equation (14) shows that present value of the sovereign varies according to the initial reported net worth and the discounted value of subsequent cash flows. To the degree that CNW is not equal to zero, a gap emerges between meeting the solvency constraint and current policy settings. If $CNW > 0$, this means that government has a positive solvency balance or buffer which could provide a funding basis for policy shifts, without recourse to increased taxation. On the other hand, if $CNW < 0$, the sovereign will have to do something different to meet its obligations.

A recurrent problem fiscal authority face is that not all assets reported in their balance sheets are up to their hand. For example, governments have declared SOE as strategic assets and have signaled not to sell those companies. Though they are reported as a positive asset in the balance sheet (affecting CNW as they generate dividends and cash flows), it is not really at government's disposal to manage their flows. The same reasoning can be used as a justification not to include some asset classes such as physical goods, forests, seacoast and cultural heritage in the RNW (thus departing from recommended GAAP standards).

Now, a method could be established to determine CNW and RNW targets, associated with the optimal level of CNW and best path for RNW. To do that, an objective function for the sovereign balance sheet management must be defined. Bradbury et al (1997) researched that a common normative mindset for the government is to maximize a social welfare function.

The sovereign makes a series of decisions relying on the specific welfare function which have, in turn, effects on its financial position. While maximizing that function economic efficiency ensures optimizing welfare as this allows policy decisions to be implemented at least cost. That is, optimal balance sheet management may be seen as including efficient management and efficient financing, tandem to monitoring asset and liability portfolio in a way which imposes the least cost on the economy as a whole. Economic efficiency is adopted as the appropriate criterion for assessing optimality of CNW. Under this approach, solving for the optimal level of CNW and the optimal path of RNW becomes a matter of solving for the most efficient sovereign financing decision (the choice between debt financing and tax financing through time). The result of that optimization problem in the simple model was then established in which it is best to set the CNW target equal to zero, to smooth the tax rate through time, and in which the RNW path is a residual of these higher order policy decisions.

The problem with that simple model is that it relies on unrealistic hypothesis. Among them, there are efficient capital markets, certainty on government's commitment to credibly follow its policy settings, no agency costs and that the financing and operating decisions are strictly separable. By relaxing those hypotheses, one would have to consider other constraints in the policy decision.

First, when $CNW = 0$ and tax smoothing is enhanced, it implicitly requires a simplification where the process of determining the expenditure stream is entirely independent of the decision as to how to finance these expenditures¹³. This is not really plausible as financing decisions are dependent on the financial position at each time t . Hence the optimal RNW path and other balance sheet variables will be disturbed influencing the intertemporal profile of the tax rate. Bradbury et al (1997) argue that specifically, these constraints are designed to minimize the sum of the inefficiencies created by deviating from a tax smoothing position and the inefficiencies in the operating decisions created by certain balance sheet positions.

Second, there are agency costs associated with the balance sheet management. As discussed in Proite (2014), it is quite challenging to align incentives and objectives of

¹³ Such an assumption is consistent with the standard separation between operating and financing decisions (eg the Fisher Separation Principle and the Modigliani Miller Irrelevance Proposition)

each relevant government segment responsible for a given asset/liability. Agency costs can be expected to arise, in large part, from the negative impact on the efficiency of decision making generated as a result of the existence of a portfolio of financial assets. To illustrate, regional government relations may limit the level of intertemporal transfers of wealth, in situations where other jurisdictions have distinct expenditure profiles but adopt a similar balanced budget tax rate strategy. The optimal results of smoothing the tax rate such that $CNW = 0$, as identified above, may dissipate to the extent that these agency problems are prevalent. The policy response is to reduce the level of agency costs expected by limiting the paths of key assets on the balance sheet defining which types of assets a given agency could hold and interference on their liabilities management from the central government views. Constraints may then be imposed which restrict the path of gross financial assets to within certain bounds, either in absolute terms or as a proportion of GDP.

Third, the sovereign may face unanticipated financial distress in a given period. This would hinder its capacity to satisfy intertemporal budget constraint consistently, imposing bounds on net debt path through time (also on RNW). Historical stylized facts show that financial shocks burdens the debtor and deteriorate credit outlook, increasing the costs to raise capital as RNW becomes negative and net debt augments. This may very well impact other elements of the balance sheet. Because sovereign credit risk profile is considered a benchmark for risk analysis for corporates, SOE and local governments, a reduced credit rating will likely to depress the level of economic activity, both in the current period and in future periods.

The implications of inserting constraints to the balance sheet will affect the intertemporal tax rate determination, but no effect on the present value of tax revenue required to be raised. Essentially, if taxes are reduced along the way the intertemporal budget constraint will not be met and, eventually, taxes will have to be raised in the future, provided debt limits will be met. The broader these constraints are, the less the impact on the setting of the tax rate. This in turn suggests a significant movement from a tax smoothing policy to something closer to a balanced budget tax rate approach [Bradbury et al (1997)].

Finally the conclusions of this discussion are that levying a precautionary amount of taxation can only be justified when the balance sheet assumes certain positions, notably with respect to the net debt position. For all other positions, no precautionary taxation can be justified. However, at some stage a deviation from the CNW target caused by shocks (perhaps over a number of periods) will require a change in the tax rate to ensure that sustainability is achieved. The point at which this occurs will be determined by an efficiency calculus. It is therefore not strictly necessary, from an economic efficiency perspective, to have liquid reserves on the balance sheet in the form of a precautionary buffer at all times, although it is understood that it would enhance market confidence. In sum, the argument is that the size of the optimal precautionary buffer is determined by the structure of the balance sheet. As a general statement the structure of the balance sheet determines the optimal precautionary buffer and thereby the optimal level of CNW.

It is noteworthy that the discussions above were led respectively by Australia and New Zealand. They enhanced the balance sheet approach and developed an interesting analytical framework capable of supporting long term policy decisions, intersecting real managerial tool (Balance Sheet) with economic theory (optimal taxation and welfare loss). This was particularly suitable to them once they had shrunk their indebtedness considerably towards very comfortable ratios¹⁴ allowing to consider other liabilities on the government spectrum and be less susceptible to unseen shocks. This paper has pointed that sort of analysis could also be employed even when debt levels are higher than the presented cases.

3- **Debt Simulation Models and the ALM approach**

One may insulate public finances from vulnerabilities that arise from macroeconomic structure susceptible to a variety of weaknesses, such as external shocks, in order to avoid having to cope with unexpected increases in costs. The objective is to create the

¹⁴ GG Debt/GDP reached 18% (2001) and 9.5% (2008) in Australia and 35% (2001) and 29% (2008) for New Zealand.

conditions to conduct proper analysis of debt portfolio¹⁵ and create the set up- financial and institutional- capable to buffer debt management in a shaky environment.

As pointed earlier, many countries commonly consider and define risk as deviations from debt servicing cash flows, examining the potential destabilizing impact of these outflows in public finances (budget, taxes and spending). Moreover, it is often the case where government revenues or spending vary in the same direction with the same financial variables that drive debt service. Hence, risk analysis envisages contrasting the impact of financial variables to debt service -the main government liability- as well as the impact on government net revenues –the main government asset, which lead the analysis to an ALM approach.

The basic idea of constructing risk models is fairly simple. It begins with modeling the liability side, the fundamental task of debt managers, and then evolves to add the assets. The most traditional assets are originated in the fiscal side (example: tax receipts incoming cash flows) and, additionally, other assets may be included such as FX-reserves. Although seemingly simple, it requires sound communication between the fiscal and monetary authorities. The reason behind this is that debt managers talk to FX-reserve managers (normally confined to the Central Bank) then align their projections on the balance of payments (BoP) behavior to formulate debt strategy, knowing the fact that FX-reserves observe different dynamics, exogenous from debt movements. Debt managers benefit from FX reserve levels, its effects on market risk and unwind issuances strategies accordingly. It is not the case where FX-reserve managers will conduct their operations upon debt management guidelines. It is the other way around and in an indirect fashion.

Coordination plays an important role here, though it is hardly the case where debt managers have total flexibility to conduct pure ALM to fully hedge their obligations. They are constrained to choose a set of strategies that are conditional on the realization of other variables which are not under their control, than adjust to it. An important distinction must be made regarding the communication across government agencies and transparency. If the Central Bank reveals its future moves regarding currency intervention and FX-reserves accumulation (say, size and currency composition), debt

¹⁵ Including contingent liabilities

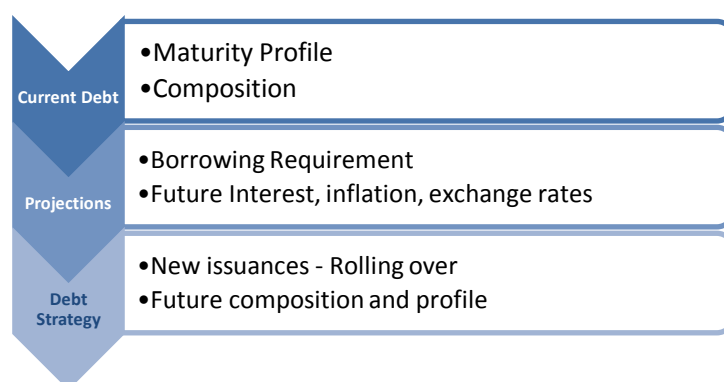
managers could anticipate that and react to implement a strategy, that is, issuing bonds with similar characteristics of the reserves. This potentially has a reassuring effect on market participants reducing risk perception. Conversely, if Central Bank's policy on foreign reserve management is opaque, debt managers could be surprised and would find it more difficult to unveil adequate debt strategies. Once more, cooperation, transparency and coordination appear to be vital to conduct ALM effectively.

On the top of it, intrinsic difficulties emerge on forecasting FX-reserve movements as it depends on random variables related to the BoP, exchange rate regime, openness of the economy, external liquidity etc. It may also be difficult to forecast as tax revenues, which is highly correlated to GDP growth¹⁶, but might be dependent on other variables such as the commodities prices, trade links etc. Having that said, the scope for analytical models to simulate assets and liabilities seem to be reduced because this particular financial asset usually is the most representative on government's portfolio. Nonetheless, after a point where reserves largely exceed short-term obligations, the benefits of increments in reserves start to be of less impact on risk perception. Thus, precise forecast on their movements tend to be of less importance if debt levels are significantly lower than this asset.

Most models start with projections on debt service path (principal and interest), based on the maturity structure (time related payments profile), debt composition (interest type related to the outstanding debt) and assumptions regarding borrowing requirements, future interest and exchange rates [Velandia, (2002)]. The current composition of the debt is known, so one must solely project the financial variables to devise future cash flows and consider the debt strategy (ongoing issuances or contracts) for accounting debt roll over in order to compute the debt servicing flows of the new loans. Figure 1 describes:

¹⁶ In turn, GDP and tax collections may also be exposed to commodities prices, international trade links, geopolitical risk etc.

Figure 1 – Inputs of a Debt Risk Model



Elaborated by the author

Because the new borrowing requirements are uncertain, future debt servicing flows depend entirely on the projections of interest and exchange rates (or such variables as inflation if the debt is inflation indexed). One can derive these projections in two fashions: (i) Deterministic; (ii) Stochastic.

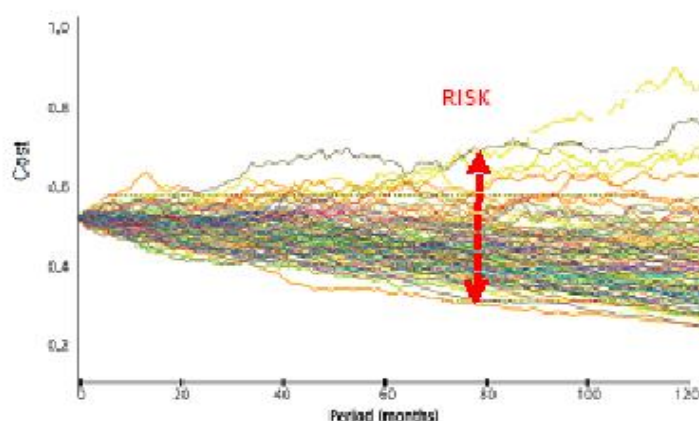
By (i), cost is determined by a baseline scenario designed to be market-neutral, whereas risk is given by debt servicing costs under various scenarios of future interest and exchange rate shocks to the baseline scenario. The nature – size, length, time – of these shocks are also created by debt managers considering their sensitivity about the economic environment in which they are inserted. For the deterministic simulation, past shocks - in the country, or in the region- can be used directly as estimates of future interest and exchange rates in worst case scenarios.

As for (ii) cost is generated by simulation techniques – such as Monte Carlo, Bootstrapping- through a large number of interest, inflation and exchange rate paths, each of which generates a specific number forecasting the upcoming debt service flow. One might choose to specify a parametric function to model these variables and to derive stochastic scenarios. Common references are the one-factor equilibrium parametrization of interest rates such as CIR, CKLS for FX-rate and GBM for inflation¹⁷.

¹⁷ CIR- Coz, Ingersoll and Ross- model. The CKLS (Chan, Karolyi, Longstaff and Sanders) model is a generalization of the CIR model. GBM- Geometric Brownian Motion. See Cabral et al (2008) and Proite (2014) for a compendium of these references.

Cost is represented by the mean of all possible scenarios and risk is measured based on any dispersion of debt servicing paths around the mean, for instance, its variance. Figure 2 depicts a hypothetical example.

Figure 2 – Description of a Stochastic Simulation of Debt Service



Source: Brazilian National Treasury and the World Bank (2010)

As can be noticed, the measures of cost and risk depend on the way future interest and exchange rates are projected, and the output of the model is as good as the assumptions used for projecting these key financial variables. In determining the cost, either through stochastic or deterministic simulation, it is essential that financial variables projections to be in line with market prices.

Velandia (2002) argues that, where liquid securities markets permit, interest and exchange rate projections should be taken from the forward markets¹⁸. The input for these “risk scenarios” is provided on the basis of historical information gathered from these markets. However, it is actually quite rare the case where the DMO has all these conditions satisfied. In most cases public debt bond market is so underdeveloped, that there are virtually no market instruments capable of providing inputs for this kind of analysis. Furthermore, it often the case in which interest rate and exchange rate are

¹⁸ Also, additional assumptions on interest rate are made. For example, well-functioning forward interest and exchange rate markets, interest parity, pure arbitrage, market neutral projections. When moving to the analysis of risk by assessing the potential impact of financial shocks on debt servicing flows, market neutrality and parity have to be abandoned.

arbitrarily fixed (exogenous), with absolute no connection with fair market values whatsoever.

To illustrate, essential rigidities might also come from the market. Aside from fixed exchange rate, market interest rates management by the monetary authority may be disconnected from market references. Regulated investors may be forced to buy securities at a given interest rate level. For example, pension funds and commercial banks have been nationalized in some Latin American countries. In Bolivia, there were two auctions in 2012, despite weekly offers from the domestic debt issuer. In Nicaragua, the same scenario was observed where few commercial banks buy short-term bills to manage liquidity. These highlight how difficult it would be to gather market parameters in the way is usually treated in these models.

When available, historical volatilities should be applied to the baseline case projections, and used as parameters of an interest or exchange rate generating process following a specified statistical probability distribution. The specification of a statistical distribution (usually assumed to be log-normal) allows the dispersion about the mean to be associated with a probability level. This permits the specification of a confidence interval for the potential increase in debt servicing costs to be used as a meaningful measure of risk [Velandia, (2002)]¹⁹.

These elements are vital to debt managers, who must use their discretion to evaluate the trade-offs existing in whether building deterministic models, priming for simplicity, or going for stochastic ones, incurring in the chance of adding unnecessary burden to the analysis.

3.1- Using Stochastic Methods to Derive Risk Measures

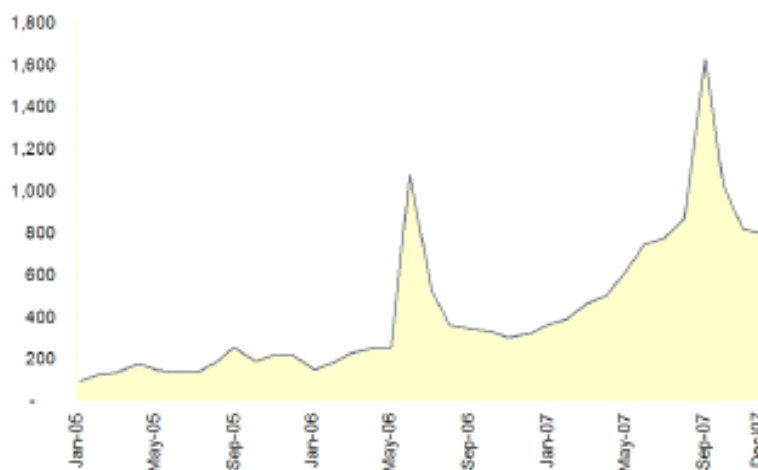
A number of risk measures can be extracted to study the effects of a given debt strategy on the overall portfolio. In essence, stochastic models such as VaR (Value at Risk),

¹⁹ Financial wire services such as *Bloomberg* and *Thompson Reuters* provide stylized shock scenarios that can be applied on given circumstances whereas normality assumptions no longer apply to historical data, subjected to disruptive market events.

CaR (Cost at Risk), CFaR (Cash Flow at Risk), BaR (Budget at Risk) are used to estimate risk measures. These are all variations from traditional VaR, that can be applied in other contexts to simulate debt levels under different states of the nature.

To give a short introduction, Value at Risk is a summary number that quantifies the worst possible loss of a portfolio at a given confidence interval, usually taken as 5%, over a given time horizon. The two parameters that can be chosen therefore are the confidence interval (usually between 95% and 99%) and the time horizon (depends on the institution but usually 1 day, with some going out to 5 days for banking purposes or even a month). A few things must be kept in mind. First, VaR is based on the parameters that are fed into the model. Decisions about data, length of time series etc must be made which will impact the VaR reported and its estimates will differ depending on these parameters. Second, time horizon – if this is too long, then the assumption of holding the portfolio constant becomes unrealistic²⁰. Hence, debt managers may wish to recalculate VaR periodically.

Figure 3 – VaR for Brazilian Debt Portfolio (BRL mn)



Source: Brazilian National Treasury

It is relatively simple to calculate the VaR for a given debt portfolio to infer the probabilistic value of the market having to absorb a public security price shock. For example, Brazilian debt managers applied this method for the fixed rate portion of the portfolio in order to assess the impact of interest rates volatility in a context where

²⁰ There is a vast literature of VaR and its variations. The reader can refer to Jorian, P.(2007).

nominal instruments were growing fast in the overall composition. The chart above shows that, in a given month, there was a 5% probability that the maximum loss would not be superior to BRL 250 mn, on average, due to increases in interest rates. It could rather be used to the overall portfolio, as illustrated in the following topic.

Using analytical VaR is the easiest method to implement when calculating worst possible loss of a portfolio at a given confidence interval. All that is needed is a time series of the assets (example: equities held by the government) /liabilities (debt bonds) prices outstanding. This could be useful for those countries that are in the early stages of accessing the market, both domestic and internationally. For instance, Paraguay, Bolivia and Honduras issued Global bonds for the first time in 2012 and have started to do VaR analysis on their portfolio.

A. Cost-at-Risk

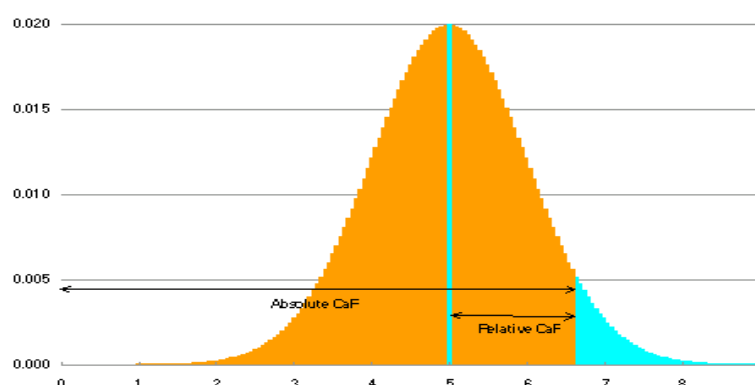
Transposing the idea above into debt costs brings the discussion back to the main context discussed in this article. Cost-at-Risk (CaR) is a supplementary measure used in the management of the interest-rate risk on the domestic central-government debt. CaR quantifies the risk on the debt and gives important input to the weighting of interest-rate risk against costs. It reflects that a central element of government debt policy providing a suitable weighting of costs against risk, when the borrowing strategy is determined [Danmarks Nationalbank, (1999)]. A distinction is made between absolute and relative CaR. Absolute CaR is the difference between the future outstanding debt, in view of a given level of significance and its initial value. Relative CaR measures the difference between this same maximum value of outstanding debt at a given significance level and the average distribution of future outstanding debt. In many instances, CaR is also used as a supplement to duration and the measure to monitor the impact of varied issuances strategies in the debt profile, under the cost-risk space.

The Danish National Bank uses the marked part of Figure 4 in the distribution where indicates the size of the costs in the 5% of cases where the costs are highest. In this case

it is found that with a probability of 95% the costs will not exceed DKK 6.7 billion, equivalent to an absolute CaR of DKK 6.7 billion (DKK: Denmark Krone). With a mean value of DKK 5 billion, relative CaR in this case is DKK 6.7 less 5 billion = DKK 1.7 billion.

Debt managers have to identify the relevant risk in order to properly measure it. Consider the case where there is maturity concentration. In a year when interest rates soar, then rolling over the debt could be a lot costlier with negative impacts on government budget. Hence, there are incentives to limit refinancing and interest rate risks distributing debt across maturities. In this sense, CaR is used as a supplement to duration, with the advantage that risk is also factored in.

Figure 4 – Danish Absolute and Relative CaR – DKK bn



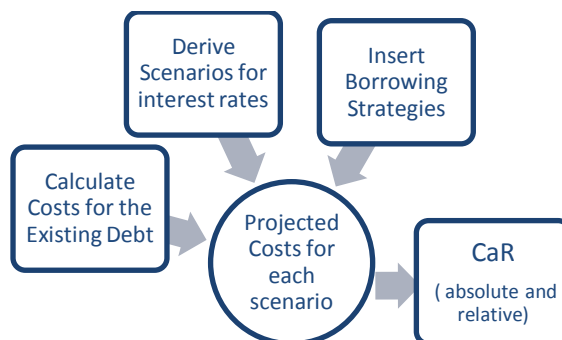
Source: Danmarks Nationalbank

To calculate the CaR while simulating the future, one would need a few inputs: (i) Because future costs depend upon the realization of interest rates, which are in turn, unknown, a set of scenarios where interest rates are simulated would result in possible future costs. When the number of scenarios and resulting costs are large enough, a probability distribution of costs is generated. (ii) The existing debt costs have to be accrued considering current payment flows. (iii) Borrowing strategy is included alongside with budget resources dedicated to debt payment. The combination of these

results in cost measures for each interest rate scenario allowing the absolute CaR computation. Then the mean value is extracted to compute the relative CaR.

The Danish National Bank proposes the following scheme:

Figure 5 – Cost-at-Risk Computation



Source: Danmarks Nationalbank, (1999). Elaborated by the author.

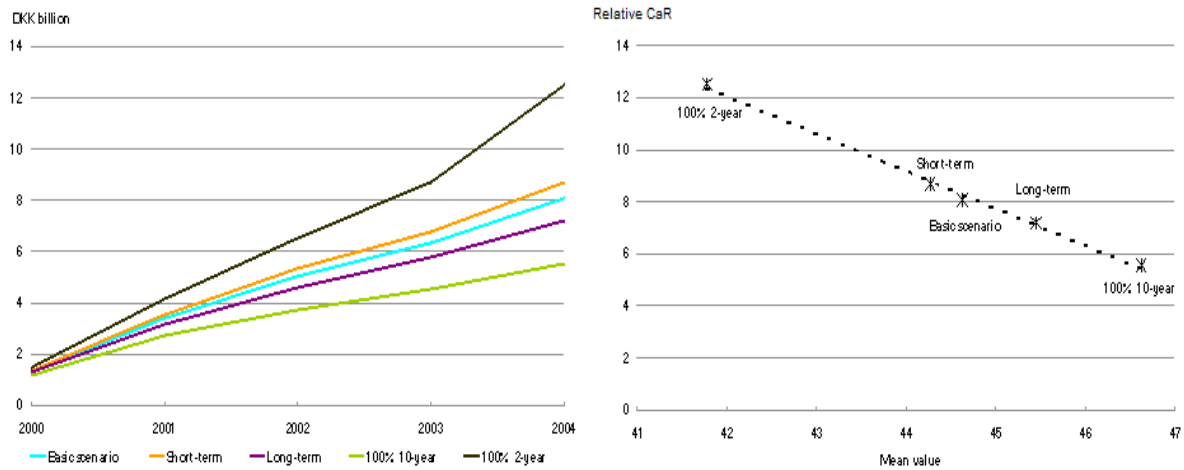
Debt managers can either use market interest rates extracted from future markets or use a term structure to forecast interest rates. In the following exercise, the Cox, Ingersoll and Ross (1985) model is used to generate various interest rates curves²¹ and devise illustrative strategies balancing 2y, 5y and 10 years nominal bonds between a basic, short-term and long term scenarios. In the basic scenario, 40% of the bonds are 2y dated, 20% are 5y dated and 40% are 10y dated. The other scenarios respectively show a 45%, 35%, 20% and 20%, 35%, 45% for 2y, 5y, 10y bonds. Two more extreme scenarios are inserted, being one with 100% of the securities issued in 10y and other with 100% issued in 2y.

The results in Figure 6 show clear differences in the values of both absolute and relative CaR for three strategies. It is also seen that the risk of incurring very high costs (long term bonds are costlier than short-term ones) is small for most strategies. This is a result of stable interest rates term structures in low levels, when compared to other countries. One underlying factor is that, as a consequence of efforts to equalize the redemption

²¹ The AFNS –Arbitrage Free Nelson-Siegel - model was introduced in 2010 in substitution to the CIR-Coz, Ingersoll and Ross- model. The main reason is that AFNS allows interest rates to be negative, a context which seem unrealistic in the early 2000's, but adequate from 2008 on, after concomitant worldwide monetary stimulus to overcome the crisis. For more on this, see Danmarks Nationalbank (2010), Litterman & Scheinkman (1991), Nelson and Siegel (1987).

profile, the debt is spread across maturity segments. All else being equal, this entails a lower risk.

Figure 6 – Relative CaR and Mean Value – DKK bn (2004)



Source: Danmarks Nationalbank

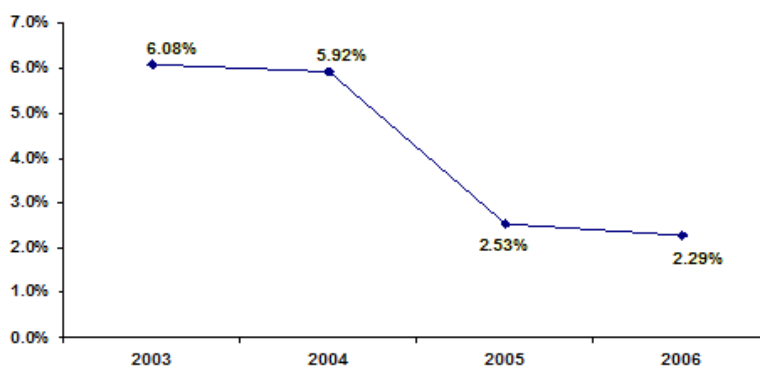
The differences in the expected costs for the three scenarios are modest in the first couple of years, but increase during the period. Based on the computed results, realistic borrowing strategies lead to stability in the expected nominal costs of around DKK 45 billion in the 5-year period. This exercise contrasts short to long-term strategies and balances costs and risks such that, for the same period, the portfolio bears an increase in relative CaR of around DKK 1.4 billion.

As for the periodicity of which these calculations are made, there is a natural requirement to consider the more short-term effect of a given borrowing strategy. The horizon for the calculations is therefore set at 5 years. It is assumed in the calculations that borrowing takes place 4 times a year, once every quarter.

In Brazil, CaR computations had been made in a monthly basis. The reason behind it resides in the existence of a more volatile debt composition and more instable markets, where interest rates in debt instruments change fast. To compute these numbers, a specific software was designed to cover some of the steps mentioned earlier. The chart below shows the results. It can be seen that, for 2006, debt costs would not raise more than 2.20% of the stock in 95% of the time. To explain, at that period prevailed a large

share of short term interest rate bonds on the portfolio, tandem to a borrowing strategy with short term nominal bonds embedded in issuance plan.

Figure 7 – Cost-at-Risk for Brazil (% of the outstanding debt)



Source: Brazilian National Treasury

Other countries use this type of calculations to assess risk. The Czech Republic's MoF quarterly publishes review to present the underlying debt portfolio risk management measures and performance with respect to the planned annual financing program and achieving the set medium-term strategic targets of debt management policy.²²

B. Cash Flow-at-Risk

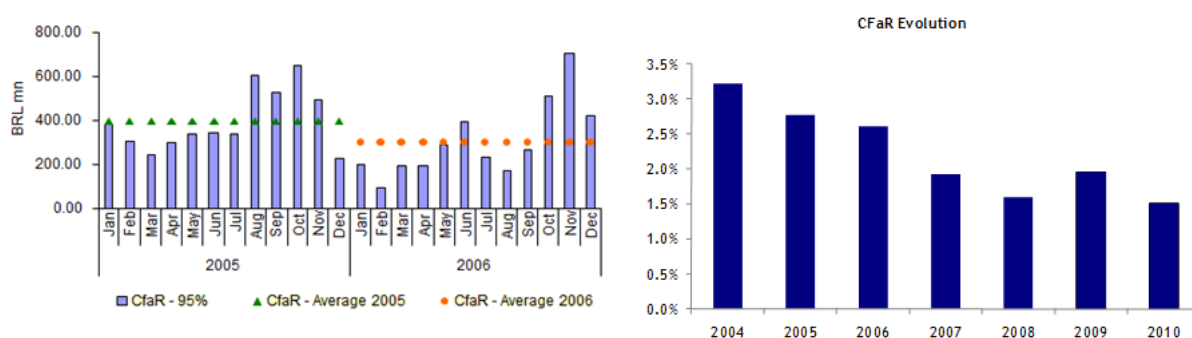
Similar rationale can be applied to address other questions related to the cash flow expected in a certain period. The Cash Flow-at-Risk (CFaR) determines, with a given significance level, the maximum payments expected in the determined future. Cashflow-at-Risk is an attempt to create an analogue to Value at Risk (VaR) that can be used by debt managers to quantify various kinds of risk exposures, including interest rate, exchange rate, and commodity price risks. [Stein et al, (2001)]. It increases the forecast ability in the budget elaboration process and assessment of public borrowing requirement, reducing the refinancing risk. Interestingly, it includes some assets and also a breakdown of the liabilities considering not only state debt, but also the derivatives portfolio and state guarantees [Czech MoF, (2007)].

²² Since 2005.

CFaR could also be used in the asset side, to help managers addressing a variety of financial decisions. For example, by providing estimates of the probability of financial distress, the CFaR method can be used in conjunction to the capital structure data to help formulate the amount of equity a government may hold in a more precise, quantifiable fashion. Moreover, it can especially helpful for those countries with the external sector based on the commodities cycle, with state owned companies figuring among an important revenues source. It can also be used to evaluate government overall risk management strategy, including the expected benefits of using derivatives to hedge commodity-price exposures or the purchase of insurance policies²³.

Figure 9 shows the outcomes of these kind of exercises for the debt portfolio in a given period in Brazil. With a 95% confidence interval, on average, the maximum amount that could be disbursed would be around BRL 370 mn (approximately USD 180 mn) during 2006. The right hand side of Figure 8 displays the CFaR in terms of the outstanding stock. That path is mostly explained because of the FX-linked rate bonds on the portfolio, though shrinking in the overall profile, was still capable of imposing cash flow risks. To offset that, the DMO bought foreign currency in advance from the market or from the official reserves at Central Bank, thus, constituting a liquidity cushion.

Figure 8 – CFaR in Brazil



Source: Brazilian National Treasury

²³ For more details, see Andr n et al (2005), Jorion (2007).

C. Budget-at-Risk

The Budget-at-Risk (BaR) measures, given a significance level, the uncertainty associated with the debt budget. BaR takes into account primary government expenditures and revenues in addition to the interest expenditure in order to smooth the budget balance. To put it differently, BaR would measure the deviations from revenues in respect to the overall expenditures and would favor debt strategies with low interest payments at times of high primary deficits, for example.

More specifically, it has commonly been used to measure the probability that the forecasted expenditures with the debt for a given year to surpass the amount approved by the legislative.

Because debt service is measured in monetary terms (cash flows), BaR is similar to CFaR, while both measure changes in uncertain future flows. By knowing the probability of exceeding the legislative authorization for the budget, debt managers may anticipate and avoid the potential gridlocks and time-consuming negotiations with the legislative for additional resources to face debt payments.

4. The Liability Side in light of the Assets Characteristics

The most straightforward way to link the modeling of the liabilities with the government's assets is by using the financial characteristics of the assets as the means to measure cost and risk for the liabilities. These metrics consist of: (a) currency denomination – the currency most closely correlated to the government cash flows of revenues and expenditures²⁴; (b) Maturity structure - the time horizon to carry out the simulation which is the basis to measure interest rate risk.

²⁴If a government issues most of its liabilities in that particular currency its budget would be less affected by changes in exchange rates and this would minimize currency risk.

At this point it is important to consider the structure of the external sector. Even if a given country may have all of its revenues and expenditures denominated in local currency, it might be still exposed to foreign exchange rate, depending on how integrated with the world its external sector is. This means that macroeconomic structure must be factored in: openness of the economy, trade links, dollarization ratio, commodity dependence so on. For instance, Latin American countries displayed a number of examples with high exposure to these factors and the external environment. Hence, it would be useful to include some measure of sensitivity of revenues to the FX-rate. For example, estimate the impact on the revenues (including asset returns) under different scenarios. Apparently, this could be very cumbersome in case of more complex economies such as the Brazilian one, but still feasible in countries with simpler economic structures.

Once local currency instruments are chosen, analogous reasoning can be applied to evaluate inflation risk associated with the monetary policy. Because revenues and expenditures might have different response to price shocks, one must understand the nature of the local economic structure and stability to assess instrument selection and decide whether to issue nominal, fx-linked or inflation-linked debt. Typically, many countries carry a positive correlation between tax revenues and inflation, while expenditures do not necessarily follow similar trend. If that is so, inflation-linked securities might find a natural hedge on the asset side. On the other hand, supply shocks, reduce the availability of goods and services and tend to shrink real government revenues. Nominal debt, in this case is not as harmful as inflation indexed debt. [World Bank, (2001)].

To this matter, the BSA may help to provide the initial clues on how an ALM could be conducted and provide useful information to build the model²⁵. The Brazilian case showed that the federal government had leeward to explore the inflation linked market without imbalances in public finances [Proite, (2014)].

²⁵ On the top of that, portfolio theory provide important argument for diversification.

As for the maturity structure, it represents another vector that should be considered in the model, as long as time horizon is relevant to measure cost and, above all, interest risk. For instance, if the government's revenues were highly responsive to short-term interest rates, (short duration or high elasticity on interest), a short term horizon would be preferred then long-term fixed rate instruments. If, on the other hand, revenues were neutral to short-term interest rates (long duration or low elasticity on interest), a long time horizon would be preferred than otherwise. In this case, the main interest rate risk would come from rolling over the debt. In fact, this would represent a positive scenario for most debt managers.

Further exploring this idea, one may ask if he could choose whether to dislocate the portfolio's duration across the yield curve without incurring in excessive costs. Unfortunately, this is not often the case due to real constraints reigning in the market structure, such as shallowness, incompleteness of the investors' base, lack of liquidity. For instance, when the sensitivity of the stream of net revenues to interest rates is known, this implies that the interest rate risk of the government debt portfolio can be measured as the volatility of debt service over a time horizon which corresponds to the duration of the government's assets (revenues). This interest rate risk will be minimized if the liability portfolio is structured to have the same sensitivity, or, duration as the assets.

Nonetheless, it is very unlikely that debt managers would have the ability to measure that sensitivity in a systematic way, other than establishing ex-post correlations. Hence, it would be troublesome to find consistency in those parameters. In practice, government's assets correlation to interest rates could be positive or negative depending on whether interest rate shocks are demand or supply driven. Since such shocks tend to be random over a long time horizon relevant to the government's objectives, that average is for revenues to have a low or zero correlation to interest rates. This is equivalent of an asset with long duration and implies a long time horizon for analyzing the risk of the debt portfolio [Velandia, (2002)]. Since in most countries, duration of the government's assets tends to be long, the time horizon should also be long. Hence, time horizon, or maturity structure emerges as a predominant aspect in the dimensions considered in the model.

In essence, the metrics for the currency and the time horizon provides good reasoning to manage the liabilities. To see that, one may be able to suppose that the flows of future fiscal surpluses the government will use to service the debt is denominated in local currency and has a long duration. It would be desirable that liabilities should bear those two main aspects. If so, the volatility of projected debt service measured in local currency over the time horizon will be low. If this ideal portfolio is not feasible, - due to market restrictions- projected debt service will be more volatile and the debtor has to deal with currency and interest rate risks. But the closer the portfolio comes to the desired metrics the lower the projected volatility and the lower the risk. Hence, by using the reference previously implied by the metrics, the simulation model allows the debtor to quantify such risks and ultimately to look for an adequate strategy to manage them.

This discussion highlights the importance of finding out the financial dynamics of the government's assets. But no matter how careful this analysis, it is possible that, in the end, some ambiguity about those true financial features remains. In fact, besides the financial aspects of the stream of official (state) revenues are stochastic, the events governing these financial features are themselves stochastic and difficult to model. That is the case of supply or demand driven shocks on the financial variables, terms of trade shocks and policy response to both type of shocks [Velandia, (2002)]. So, even if it is possible to estimate (on average) what those sensitivities are over a long time horizon, there might be periods in which those sensitivities significantly depart from their long term averages in a way that government's financial position could be at risk. Nonetheless, debt managers may investigate the composition and structure of their most significant assets and act accordingly to come up with suitable debt strategies.

For instance, take the example of international reserves, which managers tend to treat them as an exogenous asset. Moreover, central banks are ultimately responsible for managing foreign reserves and tend to choose very-low risk, highly liquid assets as investment alternatives, such as US Treasury, or German Bundes securities denominated in convertible currencies [Romanyuck, (2010)]. In the past decade, a great deal of countries have accumulated large reserves, trending to diversify part of their portfolio in a quest for high yield returns²⁶. Typically, none of this is under debt

²⁶ See Maziad et al (2011) and IMF (2010).

managers control. Despite that, they reason on the size and composition of the reserves and search a debt composition that minimizes cost and risk [Hansen, (2000)]. This could enhance market confidence in the issuer and lead them to tap in the external debt market, as well as conducting liability management operations such as switches and buy backs. Furthermore, the sole fact of reserves accumulation may have a positive impact in the domestic market and propitiates the implementation of debt strategies that would be hard to do otherwise.

In respect to that, another difficulty arises: inter-agency, or intra-governmental lack of coordination to oversee asset and liability in an integrated way, or even share the information on them. In some countries, government agencies dealing with assets (such as Central Banks, or Sovereign Wealth Funds) are not willing to cooperate with debt managers, not even on the informational ground. Thus, most debt managers tend to focus in modeling the liability side.

4.1. Including Government Assets in the Approach

Until now, the discussion has been focused on the liability side where the assets are treated as if they were exogenous. This approach has the intrinsic problem to make it more difficult to understand the financial properties of the assets, which, in turn, would calibrate the debt strategy design conditional on interest and exchange rate exposure and time-related sensitivity. The World Bank (2002) considers three ways of incorporating the assets in the analysis.

- A. Divide the government's assets into subjacent portfolios and study the financial features of these smaller parts to find natural hedges. It is similar with what's been presented in the BSA because it facilitates finding the sensitivity to interest and exchange rates for different assets and liabilities and analyzing them separately in terms of aggregating their correspondent cash flows. The underlying idea is to match the assets which carry similar financial characteristics with the corresponding liabilities in order to insulate –hedge - the overall portfolio. The costs and risks can be modeled as described earlier. This

approach reduces the problem of the indeterminacy of the assets financial features and potentially reduces the error of the risk estimate.

- B. Conduct a more explicit interaction of the simulation of the debt servicing flows with fiscal revenues and new borrowing requirements.
- C. An integrated analysis of assets and liabilities are incorporated in the model. Despite the straightforwardness of this idea, rare are the DMOs which carry this on²⁷. The idea here is to explicitly model the links between the financial variables affecting future debt service and the variables affecting government revenues.

Velandia (2002), points that one way of doing it is to build a model considering the effects of macroeconomic variables on financial variables. Specifically, that would describe domestic short-term interest rates as a function of inflation and GDP real growth respecting economic theory and predetermined correlations such as the Phillips' curve²⁸. This approach has the advantage of solving the interest rates for a given value of the macro variables guaranteeing consistency between the two.

The problem, however, is that one must select functional forms to define "reaction functions" and establish correlations between economic and financial variables that might be inappropriate. One may lack of information to perform this task. To highlight that, depending on whether the nature of a particular shock (growth pace, inflation, fx-rate movements), those correlations may change. On the top of that, there is no general equilibrium theory on how real economic-financial variables are jointly determined, which justifies the adoption of exogenous customization of these relationships.

Alternatively, one might come up with these correlations from actual data, estimating a variance-covariance matrix containing all relevant variables. Nonetheless, the problem

²⁷ The SNDO – *Swedish National Debt Office*- has attempted to do so. Its organizational structure facilitates that set up, though, with good coordination with other government agencies and fair monitoring of contingent liabilities.

²⁸ Where growth is positively correlated with higher inflation ratios.

“lies in the large number of correlation coefficients that needs to be estimated – exchange rates with different currencies, interest rates with different maturities for different currencies, and a set of macro variables for the local economy and for the relevant foreign economies”- [Velandia (2002)] because the data requirements increase largely depending on the number of unknowns for the estimates to be statistically reliable. Moreover, one must also choose a probability distribution for that variance-covariance matrix in order to allow statistical inference (construct confidence intervals) to find meaningful risk measures. A possible approach to reduce the number of coefficients to be estimated is to impose certain parameters. In essence, that is what most DMO do as it greatly simplifies this issue.

To summarize the inclusion of ALM on a more didactic way, one may consider the following: First, debt managers must know in great detail the characteristics of its debt and be able to project its future cash flows for a chosen period, including its basic financing strategy. After that, one should make inferences about the behavior of key variables such as interest and FX rates. For instance, market inputs could be used to estimate those as much as possible. As a result, this projection of debt service flows will lead to find expected cost of the strategy, which should be expressed in present value. The second step is to derive similar cost measures based on different market rates assumptions. Again, it is important to describe these cost measures in present value. The third step enhances risk estimates, in turn, defined as the difference between the cash flow given an initial scenario and the deviations around it. If one repeats this for a sufficiently large number of scenarios, it will have a distribution of costs, which can originate the mean (overall cost) and its standard deviation (risk). Some stochastic methods to measure risks are presented as follows. The fourth step is to replicate steps one to three for alternative strategies and analyze the trade-offs in the cost-risk space for each strategy. Velandia (2002) proposes the following scheme, described in Figure 9.

Aside from standard deviations of different cost measures associated with particular scenarios, other methods have been introduced to measure risk in order to supplement debt managers` sensibility to the impact of interest/FX rates changes in their portfolio.

Figure 9– Summary on Building a Risk Model for Sovereign Debt

Step 1	Step 2	Step 3	Step 4
<ul style="list-style-type: none"> •Debt service cash flows are projected forward for a specific time horizon under base case assumptions of the funding strategy and future market rates. •The base case assumptions of future market rates should be market-neutral. •The stream of debt servicing flows gives the expected cost of the debt strategy which may be expressed directly as a cash flow or converted to present value 	<ul style="list-style-type: none"> •New projections are made under alternative market rate assumptions. •These alternative cases can be generated using statistical techniques, historical analysis, worst case scenarios, etc. •These new projections deviate from the most likely path computed in step 1. •Again, the cost of these risk scenarios can be expressed as cash flow or discounted to present value 	<ul style="list-style-type: none"> •Risk could be measured as the difference between the cash flow or present value of the base case in step 1, •The range of cash flows or present values of alternative scenarios in step 2. •In stochastic simulation a measure of risk is obtained by relating a confidence level to the potential increase in debt service. •Measures of risk using confidence intervals are commonly known as VaR, or CaR. <ul style="list-style-type: none"> •VaR deals with the volatility of the present value of the debt, •CaR deals with the volatility of future debt servicing flows, and express the maximum expected increase in annual costs relative to the mean with a given probability on a given period. 	<ul style="list-style-type: none"> •Do 1 to 3 again for alternative strategies •Analyze the trade-offs existing between costs and risks for each strategy

Source: Velandia (2002). Elaborated by the author.

5. Conclusions

This article showed how some countries could use a risk model for building a more analytical tool to understand the size and sources of risk exposures the debt may show, depending on the characteristics such as maturity profile and composition. In a context where debt managers seek to modernize their analysis, the models presented above could be a starting point for implementation. While optimization of balance sheet or net worth are more affine to optimal taxation theory, debt service simulation models predominate in most DMOs.

Several limitations arise when adopting the latter. One of the most important is the recommendation where debt managers should rely on the usage of market indicators for deriving important parameters. In most low-income countries, debt managers usually deal with constrained or even dysfunctional markets, making it very difficult to derive

such parameters. Another important restriction is the fact that contingent liabilities are disregarded, but often surge in their reality with big impacts on debt levels. Despite that, its simplifications allow the government to focus on financial variables endogenously, adding the possibility to simulate scenarios for interest and exchange rates besides their interactions. Other important variables such as revenues flow could be treated exogenously, allowing the government to devise independent paths from the simulation of liabilities, which could strengthen the verification of diverse debt strategies.

Last, it allows the possibility of including the asset side in the analysis. The idea of breaking up the government assets in different sets of sub-portfolios facilitates the sensitivity analysis to changes in interest and exchange rates. Besides facilitating its implementation it simplifies and minors errors in risk estimation.

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